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HYDROLOGICAL MEMORANDUM 42

Quintiles of the dates of return to, and loss of, field capacity and of hydrologically effective precipitation over England and Wales

by

R. C. Tabony, B.Sc.

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LIST OF SYMBOLS AND ABBREVIATIONS USED

C_v	Coefficient of variation of annual rainfall (the standard deviation divided by the mean)
D_l	Date of loss of field capacity (the last date on which SMD is zero prior to reaching a value of 12.7 mm (0.5 inches) for the first time in spring
D_r	Date of return to field capacity (the first date on which SMD is zero after the year's maximum SMD has been attained)
D_r'	Estimated value of D_r from Figures 5 and 6
D_r''	Corrected D_r', taking account of $P_s - P_s'$
D_{rb}	The date D_r obtained in the Meteorological Office hydrometeorological bulletin.
E_p	Potential evapotranspiration
$\bar{E}_p$	Average potential evapotranspiration
G	Increase in $R_{e,75}$ per mm decrease of $(\bar{R} - \bar{E}_p)$ for the summer half-year
H	Observed $(\bar{R} - \bar{E}_p)$ for the summer half-year less the Table 10 value of the summer half-year $(\bar{R} - \bar{E}_p)$
P_s	Average summer half-year rainfall divided by $\bar{R}$
P_s'	Estimated value of P_s from Table 4
R	Rainfall
$\bar{R}$	Average rainfall
R_e	Hydrologically effective precipitation (the rain-water surplus after any soil-moisture deficit has been removed and evapotranspiration has been allowed for)
$R_{e,75}$	R_e calculated for a 75-mm root constant
$R_{e,200}$	R_e calculated for a 200-mm root constant
SMD	Soil-moisture deficit (the depth of water required to restore a soil sample to field capacity, i.e. holding the maximum amount of water possible against gravitational drainage)

HYDROLOGICAL MEMORANDUM No. 42

QUINTILES OF THE DATES OF RETURN TO, AND LOSS OF, FIELD CAPACITY AND OF HYDROLOGICALLY EFFECTIVE PRECIPITATION OVER ENGLAND AND WALES

by R. C. Tabony, B.Sc., Meteorological Office

SUMMARY

A method is presented for estimating from readily available data the distribution of several hydrological parameters for England and Wales. For root constants of 75 mm and 200 mm, mean values of the annual and seasonal differences of rainfall and potential evapotranspiration are used to estimate quintiles of the dates of return to, and loss of, field capacity, and the totals of hydrologically effective precipitation for the hydrological year and for each of three periods within the year. The first period starts on 1 June, the second with the date of return to field capacity, and the third with the date of loss of field capacity.

1. INTRODUCTION

1.1 *Preface*

The hydrologically effective precipitation (R_e) in a given period is defined as the rain-water surplus after evapotranspiration (hereinafter referred to merely as evaporation) has taken place and any soil-moisture deficit (SMD) has been removed. R_e is the amount of water available for run-off to rivers and reservoirs and/or percolation to aquifers. R_e is extremely seasonal in amount. It is normally greatest in those winter months when the soil is at field capacity. A soil is said to be at field capacity (see Smith, 1971) when it holds the maximum amount of water possible against gravitational drainage. In the present study, the date of return to field capacity (D_r) is defined as the first date on which the SMD is zero after the year's maximum SMD has been attained. The date of loss of field capacity (D_l) is defined as the last date on which the SMD is zero, prior to reaching a value of half an inch (12.7 mm) for the first time in the spring.

There are considerable variations in the annual totals of R_e at any place and wide variations in its time of occurrence. R_e occurs between D_r and D_l but year-to-year variations in these dates, particularly of D_r, are considerable. Hydrologists are chiefly interested in the annual total of R_e since this represents the amount of water from which surface and subterranean resources are replenished. Agriculturists and civil engineers are more interested in D_r and D_l. Given long records of daily rainfall and monthly potential evaporation, it would be possible to obtain D_r, D_l and R_e directly. Long records are available for only a small number of stations and it is useful to have a means of estimating the variability of R_e for any location from mean rainfall and mean potential evaporation.

To provide a complete description of the variability of R_e, the year is divided into three periods:

(1) From 1 June until D_r. During this period R_e is usually zero.

(2) From D_r until D_l. During this period evaporation and SMDs are both small and most precipitation which falls is hydrologically effective.

(3) From D_1 until 1 June. Mean evaporation is beginning to exceed mean rainfall, but only small SMDs are usually present and so R_e occurs in some years.

In this paper, quintiles of D_r, D_1 and R_e are calculated for the year as a whole and for the three periods. The chief factors influencing or controlling D_r, D_1 and R_e are:

(a) average annual rainfall,
(b) average annual potential evaporation,
(c) standard deviation of annual rainfall, and
(d) average seasonal distribution of rainfall.

Items (a) and (b) are obvious variables; items (c) and (d) will be discussed later.

Smith (1971) has already presented an analysis of D_r and of winter and spring R_e. He relates D_r and R_e only to annual $\bar{R}$; in the present study the effects of potential evaporation (E_p) and of the seasonal distribution of rainfall are also included. The method of calculation of SMD is that employed by Grindley (1967) in the hydrometeorological bulletin of the Meteorological Office. This method differs from that used by Smith (1971); these differences are discussed later.

1.2 *Methods*

The results obviously depend upon the method used to calculate SMD. In this paper a water surplus, available for run-off and/or percolation, is assumed to occur only when the soil is at field capacity. This assumption is common to most simple SMD models.

The potential evaporation is defined as that evaporation which occurs when water is non-limiting. The potential SMD is that SMD which would exist if evaporation always proceeded at the potential rate. To calculate actual SMD, allowance must be made for the reduction in the rate of evaporation below the potential rate as soil-moisture stress develops. In the hydrometeorological bulletin of the Meteorological Office the concept of root constant (Penman, 1949) is used for this purpose. All the soil moisture lying within the rooting depth of a plant is considered to be transpired at the potential rate; this amount of water, expressed as equivalent depth of rainfall, is the 'root constant'. In Penman's model a further 0.8 inch (20.3 mm) of water is then allowed to be transpired at the potential rate before the ratio of actual to potential evaporation falls to a value of one-twelfth. In this study, root constants of 75 mm and 200 mm, generally considered appropriate to grass and trees respectively, are used.

Smith (1971) uses a soil-moisture model also representative of grass. Smith's model and that for a 75-mm root constant are compared in Figure 1. The two models may be expected to yield a similar relationship between actual and potential SMD for potential SMDs less than 50 mm and greater than 200 mm, but to differ by about 25 mm for potential SMDs around 100 mm. The usefulness of the calculated values of D_r and R_e from the above models would be much increased if they could be compared with observed values. Unfortunately very few reliable observations of D_r and R_e are yet available.

1.3 *The data*

Daily computed values of SMD for 75-mm and 200-mm root constants are available for 74 stations from 1941 onwards. The stations used are mapped in Figure 2, together with their average annual rainfalls for the period 1916-50. The network is thin in the south-west peninsula, in Wales, and in northern England, but there is a fairly good coverage over the rest of England. Averages of rainfall and evaporation for 1941-66 should have been used but were not available. In lieu of these the rainfall averages used are for the period 1916-50 and the averages of potential evaporation were obtained from values provided by Smith (1967) for the period 1950-64, these being the only ones available. Data for Kew from 1876 show that the standard deviation of 15-year potential evaporation totals is 2 per cent, while the standard deviation of a 30-year-average annual rainfall is around 3 per cent. Hence the use of disparate periods for averages of rainfall and potential evaporation is unlikely to have had a significant effect on the results obtained.

For the 25 years 1941/42 to 1965/66 the data from the 74 stations were used to calculate values of D_r, D_l and R_e. R_e was also calculated separately for the periods bounded by D_r, D_l and 1 June. The root constants are nominally for 75 mm and 200 mm but, for convenience, computations were performed using root constants of 3 inches (76.2 mm) and 8 inches (203.2 mm). The values of D_r, D_l and R_e for each station were ranked and separated into five equiprobable classes, using the scheme shown in Figure 3. For D_r and D_l the class boundaries were estimated and are referred to as 'quintile boundary dates'. For R_e, mean values of each of the five classes were estimated and are referred to as the 'quintiles of R_e'. These mean values were chosen in preference to the quintile boundaries of R_e because their irregular fluctuations from station to station were smaller.

1.4 *Variability of annual rainfall*

The ranges of values of D_r and R_e obviously depend upon the local variabilities of annual rainfall. The coefficient of variation of annual rainfall (C_v) generally increases with decreasing annual $\bar{R}$, but the range of values of C_v is only from about 17 to 13 per cent of annual $\bar{R}$, and the average C_v for a given station obtained from 30 years of data is subject to a standard error of 1.5 per cent. Hence geographical variations in C_v are only taken into account implicitly through the inverse relationship between C_v and annual $\bar{R}$.

1.5 *Seasonal distribution of rainfall*

The main feature of the average seasonal distribution of rainfall in Britain is the mean difference between the summer and winter half-year contributions, the former being defined as April-September. In the west and extreme south there is a pronounced maximum of winter rain, while in the east there is a slight summer maximum. To see how these differences affect D_r consider three stations, A, B, and C all having annual average rainfall of 660 mm and potential evaporation of 480 mm. Station A has rainfall uniformly distributed throughout the year, station B has a summer maximum of rain, and station C a winter maximum of rain. Consider a dry year and a wet year in each of which the ratios of seasonal to annual rainfall are preserved and (for simplicity) evaporation always occurs at the potential rate. Table 1 presents schematic monthly totals of R and E_p which enable end-of-month SMDs to be calculated.

From these figures estimated dates of return to field capacity are quoted. In the dry year, field capacity is attained on 31 March at all three stations, whereas in the wet year D_r ranges from early September to early November. The early returns to field capacity vary with the proportion of rain falling in the summer half-year, while the late returns to field capacity are independent of it. The relevant summer half-year rainfall is that which falls between D_l and D_r. Hence the effect of the summer half-year rain diminishes from the early to the late returns to field capacity.

2. DATES OF LOSS OF, AND RETURN TO, FIELD CAPACITY

2.1 *Date of loss of field capacity in relation to rainfall and potential evaporation*

Between 1941 and 1966, D_l (as calculated for the routine bulletin issued by the Meteorological Office) nearly always occurred within the inclusive period February to April, and so it was decided to correlate the quintile boundary dates of D_l with the average (rainfall minus potential evaporation), or $(\bar{R} - \bar{E}_p)_{F-A}$, for February-April inclusive, for the 74 stations, to provide a model for predicting the distribution of D_l. Linear regression lines fitted by the method of least squares are displayed in Figure 4. Q_{12}, Q_{23}, Q_{34} and Q_{45} refer to the first-second, second-third, third-fourth, and fourth-fifth quintile boundary dates respectively. Q_E and Q_L refer to the earlies and latest dates in the 25-year sample. Some idea of the confidence limits can be gained from the inclusion of the data with the original (bulletin) quintile boundaries in Figure 4. The actual errors involved (i.e. as between the bulletin and the present model) are given in the left-hand column of Table 2. The root-mean-square differences were calculated in each case using the difference between the date given in the hydrometeorological bulletin of the Meteorological Office, referred to as the 'bulletin date', and the date calculated from the regression lines in Figure 4, referred to as the 'calculated date'. It can be seen from the graphs in Figure 4 that the estimation of root-mean-square errors independent of $(\bar{R} - \bar{E}_p)_{F-A}$ is reasonable.

2.2 *Date of return to field capacity based on annual* $(\bar{R} - \bar{E}_p)$

To obtain a model for the distribution of D_r, quintile boundary dates of D_r were related to annual $(\bar{R} - \bar{E}_p)$ by means of cubic regression lines fitted by the method of least squares. These were the least complicated regressions that gave a good fit and, together with the data and the original (bulletin) quintile boundary dates, they are displayed in Figures 5 and 6. At stations where an annual $(\bar{R} - \bar{E}_p)$ exceeds about 400 mm, the dates of return for 75-mm and 200-mm root constants are very similar, mainly due to the summers being sufficiently wet for evaporation to proceed at the potential rate from vegetation types spanning both root constants. As $(\bar{R} - \bar{E}_p)$ decreases, the evaporation becomes progressively less, and D_r occurs progressively earlier, for a 75-mm than for a 200-mm root constant.

For the 75-mm root constant, out of 1850 station-years examined, only five cases were found when SMD persisted through the winter. This occurred much more frequently for the 200-mm root constant, however, and Table 3 gives the probability of non-return to field capacity related to annual $(\bar{R} - \bar{E}_p)$. Table 3 shows that, in the driest parts of the country, field capacity for a 200-mm root constant is only returned in 55 per cent of years, whereas for annual $(\bar{R} - \bar{E}_p)$ exceeding 160 mm the proportion exceeds 90 per cent.

2.3 *Date of return to field capacity taking into account proportion of summer half-year rain*

It has been indicated that improved estimates of D_r can be made by taking into account the proportion of the annual rain which falls during the summer half-year. Changes of up to 30 days in the estimate of D_r may result. To some extent differences in the proportion of rain falling in the summer half-year (April-September) have already been implicitly incorporated into the data fitted by the regression lines of Figures 5 and 6. This is because most of the dry stations are in eastern England, where the rain is fairly evenly distributed throughout the year, whilst most of the wet stations are in the west where there is a winter maximum of rain.

If the value of the fraction (P_s), the mean summer half-year rainfall divided by the mean annual rainfall, is plotted against the annual ($\bar{R} - \bar{E}_p$) and a quadratic regression line is fitted by the method of least squares, the figures presented in Table 4 are obtained. It is not the absolute value of P_s which is of interest as this has already been accounted for in Figures 5 and 6: the interest lies in the departure of the individual station values of P_s from the value of P_s corresponding to the annual ($\bar{R} - \bar{E}_p$) in Table 4.

Let P_s' be the estimated value of P_s from Table 4.

Now let D_{rb} be the observed D_r as recorded in the bulletin, and let D_r' be the estimated D_r from Figures 5 and 6.

Let D_r'' be the corrected date of return to field capacity, taking into account $P_s - P_s'$.

Let $f(D_r') = (D_{rb} - D_r')/(P_s - P_s')$ be estimated from a regression of $D_{rb} - D_r'$ on $P_s - P_s'$.

Now $$\begin{aligned} D_r'' &= D_r' + (\text{error in } D_r') \\ &= D_r' + (D_{rb} - D_r') \\ &= D_r' + f(D_r')(P_s - P_s'). \end{aligned}$$

The quintile boundary dates of D_{rb} were categorized according to the calendar month in which D_r' fell. For each month, graphs of $(D_{rb} - D_r')$ against $(P_s - P_s')$ were plotted. Straight lines were fitted by the method of least squares through the scatters of points, and the gradients of these lines were taken as the value of $f(D_r')$. The straight-line relationships were good, with correlation coefficients around 0.85. A little smoothing, commensurate with the degree of correlation, was used to obtain the curves of $f(D_r')$ presented in Figure 7. The predominantly negative values of $f(D_r')$ imply that for a station with more summer rain than that given in Table 4 (positive $(P_s - P_s')$), D_r is earlier than indicated by Figures 5 or 6 (D_r'' earlier than D_r').

In Figure 7 consider first the curve for the 200-mm root constant. Here evaporation proceeds at the potential rate for much of the time, and so D_r depends upon $R - E_p$ from about April until D_r occurs. The effect of April-September summer rainfall is therefore at a maximum when D_r occurs in September. When D_r occurs in March, the winter half-year rain is just as important as the summer half-year rain and the proportion of rain falling in the summer is not important. It will be recalled that in section 1.5 it was concluded that early returns to field capacity depend upon P_s, whereas late returns are independent of it.

The curve in Figure 7 for the 75-mm root constant diverges from that for the 200-mm root constant because evaporation for the lower constant is more often limited by soil-moisture stress. For the 75-mm root constant, evaporation proceeds at the potential rate during the wet summer but at a very much reduced rate for much of the dry summer. The differences in SMD between the dry and the wet summers is much less for the 75-mm root constant than for the 200-mm root constant. Hence the effect of the summer half-year rain is less for the 75-mm root constant than for the 200-mm root constant. This is illustrated in Figure 7 by the way in which the curve for the 75-mm root constant climbs away from that for the 200-mm root constant. By the end of November, winter rainfall has more than counterbalanced the effect of summer rainfall.

2.4 *Procedure for estimating dates of return to field capacity*

The procedure for estimating the quintile boundary dates of return to field capacity is therefore as follows:

(a) Calculate the value of $(P_s - P_s')$, P_s' being obtained from Table 4.

(b) Obtain the first estimate D_r' of the quintile boundary dates from Figures 5 and 6.

(c) Obtain the values of $f(D_r')$ from Figure 7 according to the value of D_r'.

(d) Calculate the corrected value D_r'' from the relation $D_r'' = D_r' + f(D_r') \cdot (P_s - P_s')$.

As an example, the results for Worthing and Rothamsted are compared for a root constant of 75 mm. Both stations have a similar value of annual $(\bar{R} - \bar{E}_p)$, but whereas the rainfall is fairly evenly distributed throughout the year at Rothamsted, there is a pronounced maximum of winter rain at Worthing. The steps in the calculation are set out in Table 5. A comparison of the results for both stations shows, not unexpectedly, that in view of their similar annual $(\bar{R} - \bar{E}_p)$, the first estimates D_r' are within two or three days of one another. After allowance has been made for the extra contribution of the summer half-year rain at Worthing, however, the date of return to field capacity is significantly delayed.

Estimates of the errors associated with the calculated D_r are given in Table 2. Where the errors of D_r based on annual $(\bar{R} - \bar{E}_p)$ alone are large, taking P_s into account leads to little or no improvement, but where the errors are less, appreciable improvement is obtained.

3. HYDROLOGICALLY EFFECTIVE PRECIPITATION

3.1 *Quintiles of R_e over the whole year*

If evaporation always occurred at the potential rate, and field capacity was attained every year, the quintile values of annual R_e would increase linearly with annual $(\bar{R} - \bar{E}_p)$. Deviations from these simple relationships occur for two reasons:

(a) evaporation falling below the potential rate under conditions of soil-moisture stress. The reduction in evaporation leads to smaller SMDs than would be expected from evaporation at the potential rate during the summer, an earlier than expected D_r, and hence an increase in annual R_e; and

(b) non-return to field capacity. Suppose a year in which field capacity is not attained has a maximum SMD of 50 mm. Suppose that in the following year the rainfall exceeds the evaporation by 50 mm. The values of rainfall minus evaporation ($R - E$) for each year are -50 mm and +50 mm respectively. The R_e in each year will, however, be zero, the 50-mm deficit of ($R - E$) in the first year being carried over and eliminating the 50-mm surplus of ($R - E$) in the second year. This effect results in frequent occurrences of near-zero values of R_e.

For the 75-mm root constant, non-return to field capacity occurred in only 5 of the 1850 station-years examined, while limited evaporation due to soil-moisture stress was common, the frequency of limited evaporation increasing with decreasing annual ($\bar{R} - \bar{E}_p$). Quintile values of annual R_e for the 75-mm root constant ($R_{e,75}$) are related to the annual ($\bar{R} - \bar{E}_p$) in Figure 8, where both regression lines and the original (bulletin) data are displayed. The concave shape of the regression lines, which were fitted by eye, is caused by the difference between potential and actual evaporation increasing progressively more rapidly as the annual ($\bar{R} - \bar{E}_p$) decreases (i.e. for the drier and more 'potentially evaporative' stations).

Quintile values of annual R_e for the 200-mm root constant ($R_{e,200}$) are related to the annual ($\bar{R} - \bar{E}_p$) in Figure 9. The linear portions of the curves, (fitted by least-squares linear regression) for annual ($\bar{R} - \bar{E}_p$) greater than about 300 mm, apply where evaporation always occurred at the potential rate and field capacity was always attained. The drier portion of the quintiles (fitted by eye) begin to converge towards a zero value of hydrologically effective precipitation before ($\bar{R} - \bar{E}_p$) reaches zero, mainly because of the effect of non-return to field capacity.

3.2 *Quintiles of R_e from D_r to D_1, D_1 to 31 May, and 1 June to D_r*

The quintiles of R_e from D_1 to 31 May are related to the annual ($\bar{R} - \bar{E}_p$) in Table 6. The quintiles are generally the same for both 75-mm and 200-mm root constants, a result of D_1 being identical for both. For the drier stations, however, the higher quintile values of hydrologically effective precipitation for the 200-mm root constant fall below those for the 75-mm root constant. These differences arise mainly from those years in which field capacity was not attained for the 200-mm root constant but when some R_e occurred for the 75-mm root constant.

The quintiles of R_e from 1 June to D_r are related to ($\bar{R} - \bar{E}_p$) for the summer half-year in Table 7. The quintile values are the same for both 75-mm and 200-mm root constants. This is because those summers in which R_e occurred were sufficiently wet to ensure that, prior to the occurrence of R_e, evaporation proceeded at the potential rate for both root constants.

The quintile values of R_e from D_r to D_1, i.e. mainly winter R_e, for the 75-mm and 200-mm root constants are related to the annual ($\bar{R} - \bar{E}_p$) in Tables 8 and 9 respectively. The relationships are similar to those for the whole year, the quintile values being less than those of the annual totals by the relatively small amounts falling between D_1 and D_r.

3.3 *Corrections to quintile values of $R_{e,75}$ due to proportion of summer half-year rain*

For the 75-mm root constant, quintile values of R_e for the whole year do not linearly relate with annual $(\bar{R} - \bar{E}_p)$ because of limited evaporation during periods of soil-moisture stress which may occur during the summer half-year, i.e. in a dry summer the moisture stress is high, actual evaporation is reduced well below current E_p, and consequently annual R_e is increased. Section 3.4 describes a procedure for estimating values of $R_{e,75}$ in which the proportion of rain which falls in the summer half-year is taken into account.

3.4 *Procedure for estimating quintile values of $R_{e,75}$ over the whole year*

The procedure for calculating quintile values of $R_{e,75}$ over the whole year is set out in Table 12. The first step is to obtain the quintiles based on annual $(\bar{R} - \bar{E}_p)$ from Figure 8. The summer half-year $(\bar{R} - \bar{E}_p)$ associated with the annual $(\bar{R} - \bar{E}_p)$ is obtained from Table 10, and this value is subtracted from the observed $(\bar{R} - \bar{E}_p)$ for the summer half-year to give H. The value of G, the increase in $R_{e,75}$ per 1-mm decrease of $(\bar{R} - \bar{E}_p)$ for the summer half-year, corresponding to the annual $(\bar{R} - \bar{E}_p)$ is derived from Table 11, and the product GH is subtracted from the set of quintiles based on annual $(\bar{R} - \bar{E}_p)$ to give the final set of quintiles. Table 12 shows that for Rothamsted the final quintiles are close to the first estimates, while for Worthing the first set of quintiles have been increased by 52 mm. It can be seen that the final set of calculated quintiles lie closer to the bulletin values than the first set.

A guide to the accuracy of the calculated totals of R_e is given in Table 13.

3.5 *Comparison of results with those of Smith (1971)*

A comparison between the results of the present study and those of Smith (1971) is presented in Table 14. The comparison is made between the two 'grassland' soil-moisture models for Worthing and Rothamsted. Good but not exact comparisons of R_e can be made by equating the amounts received between D_r and D_1 with the amount received between D_r and 31 March. The figures for D_r and R_e are in broad agreement with one another. As Worthing and Rothamsted both have an annual $\bar{R}$ of 700 mm, Smith (1971) obtains identical results for both stations. A contribution of the present study is that it introduces differences between the stations by virtue of their differing seasonal distributions of rainfall.

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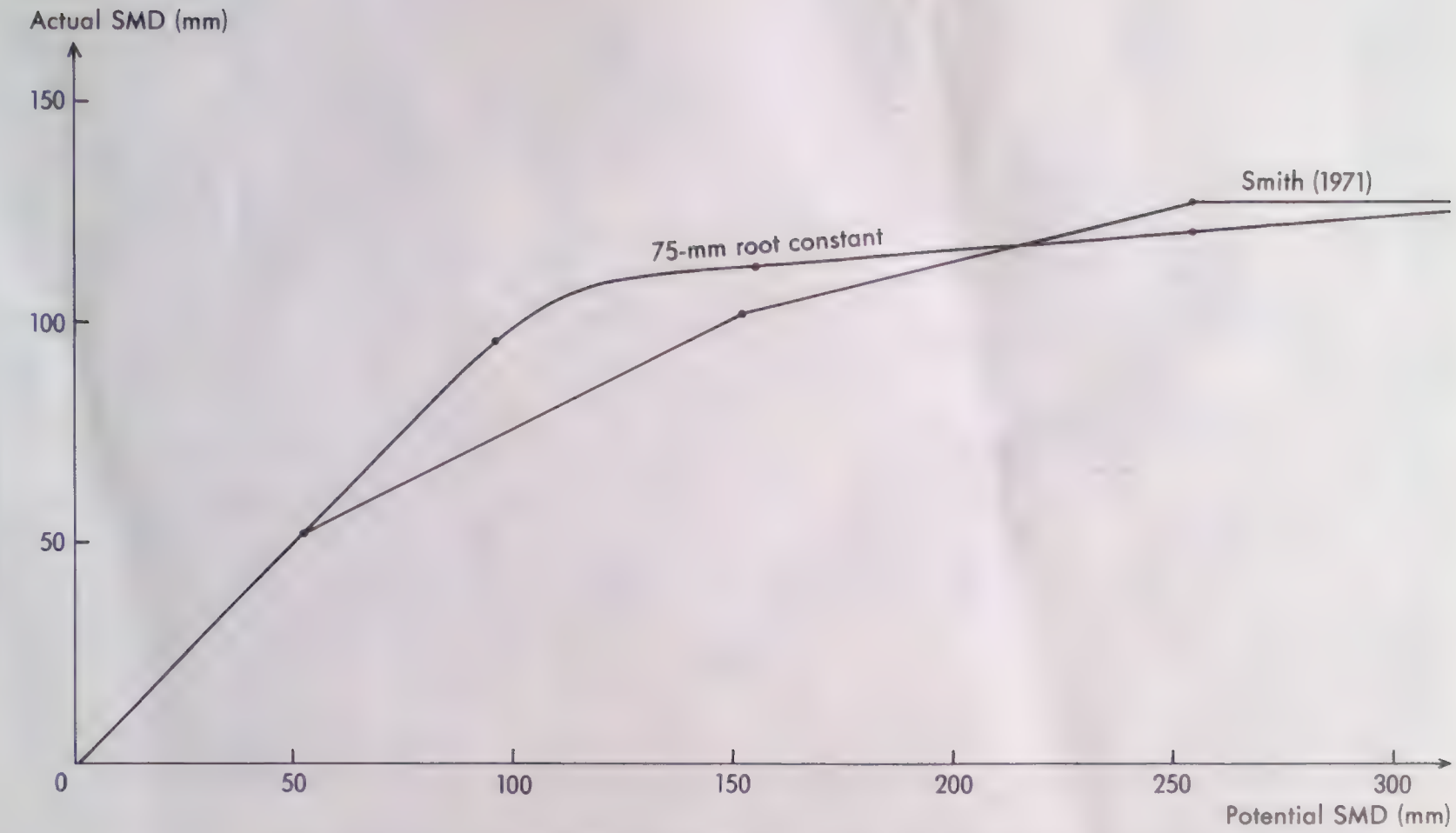

Figure 1. Comparison of two grassland soil-moisture deficit models: (a) that used in this study (75-mm root constant), and (b) that used by Smith (1971)

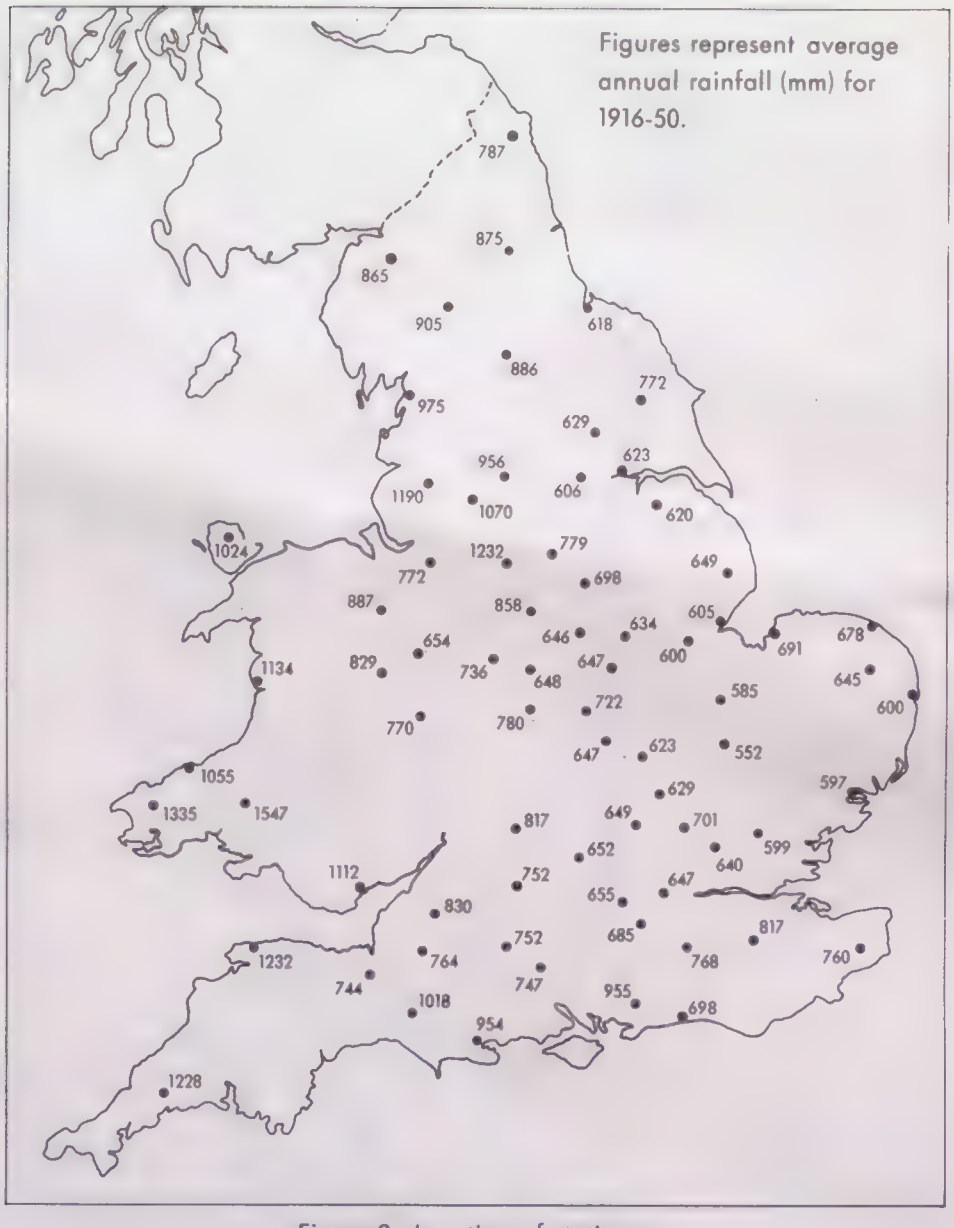

Figure 2. Location of stations

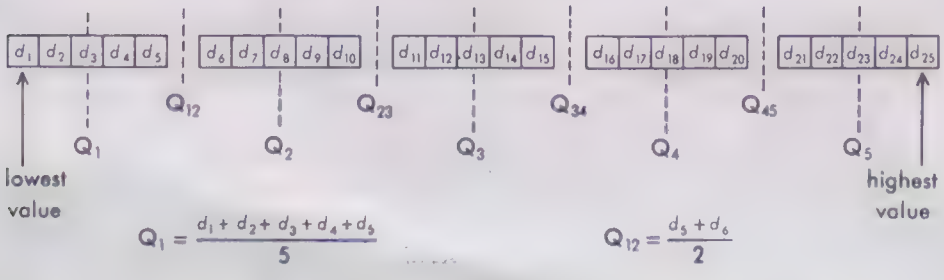

Figure 3. Quintiles (Q_1 to Q_5) and quintile boundaries (Q_{12} to Q_{45}) of the 25 years (1941-66) of ranked values (d_1 to d_{25}) of R_e, D_l and D_r

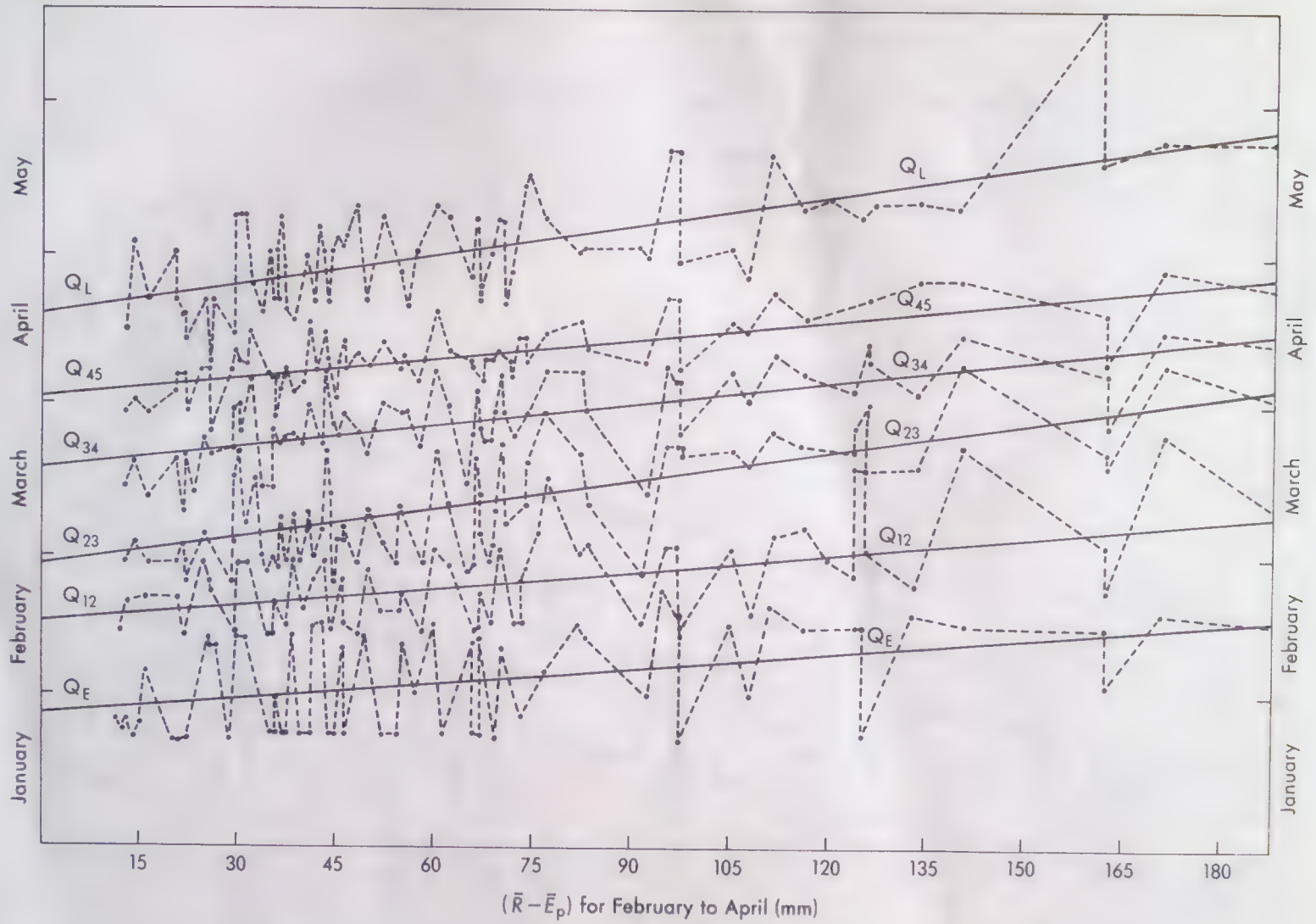

Figure 4. Date of loss of field capacity related to $(\bar{R}-\bar{E}_p)$ for February to April

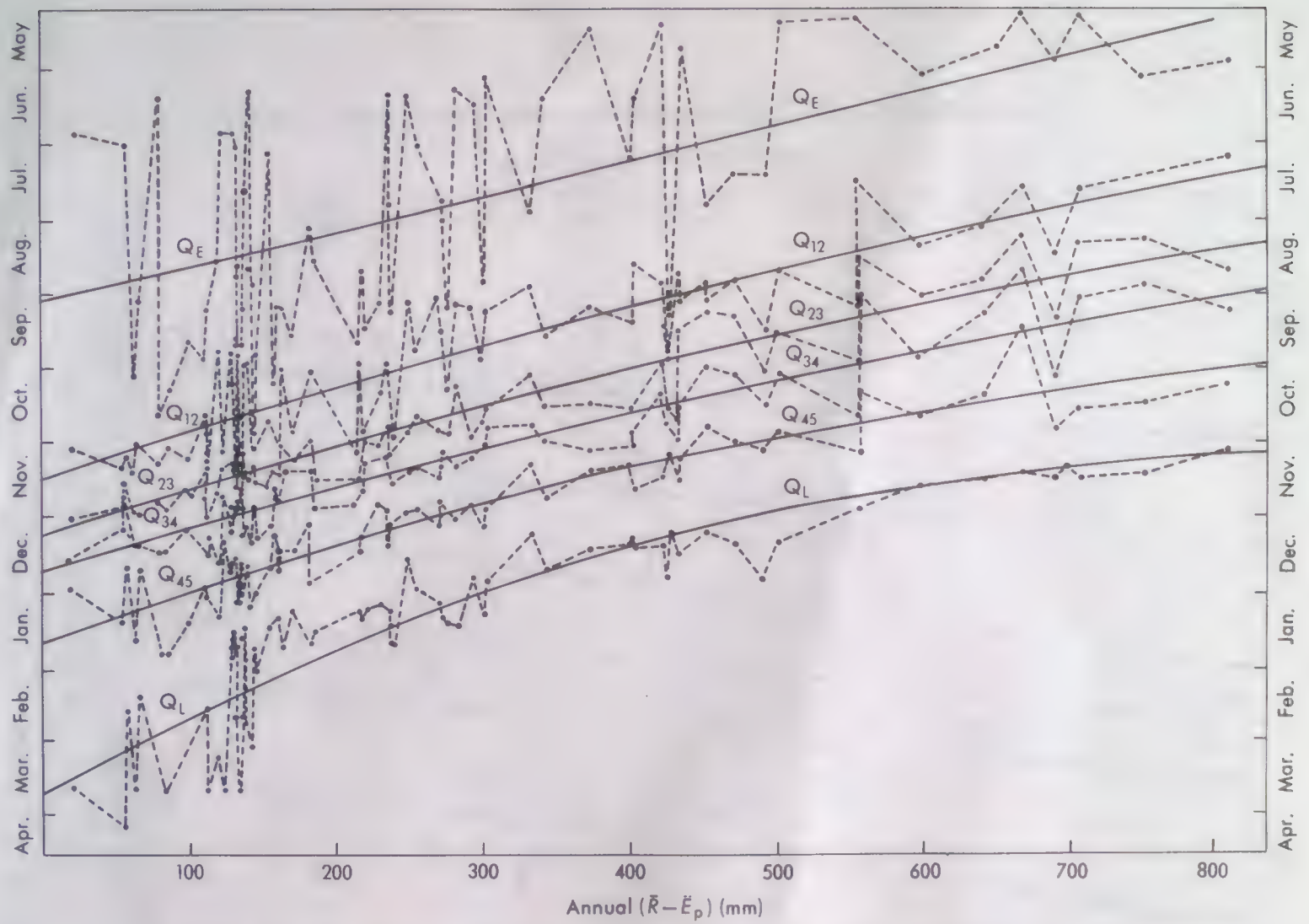

Figure 5. Date of return to field capacity for 75-mm root constant related to annual $(\bar{R}-\bar{E}_p)$

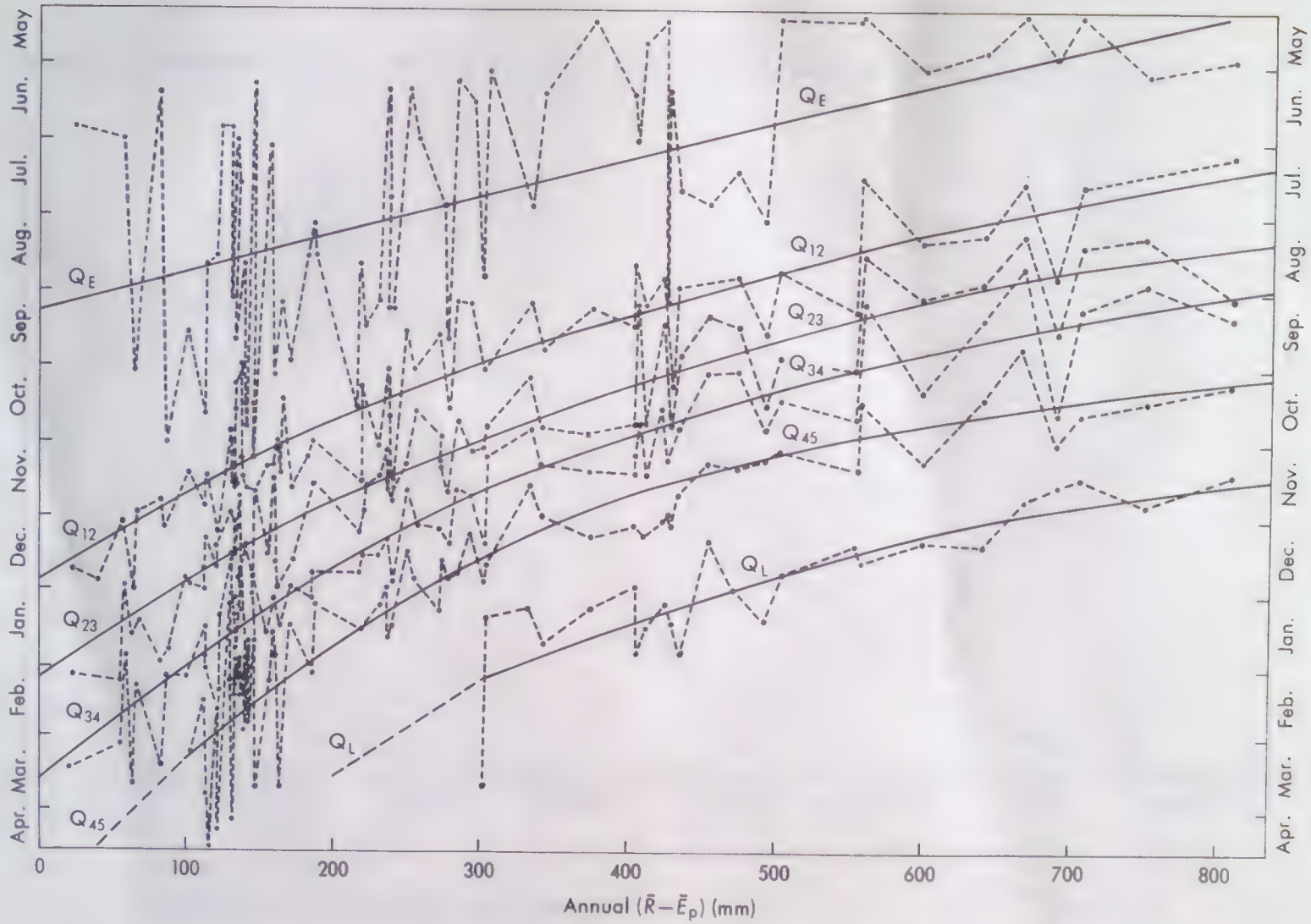

Figure 6. Date of return to field capacity for 200-mm root constant related to annual $(\bar{R}-\bar{E}_p)$

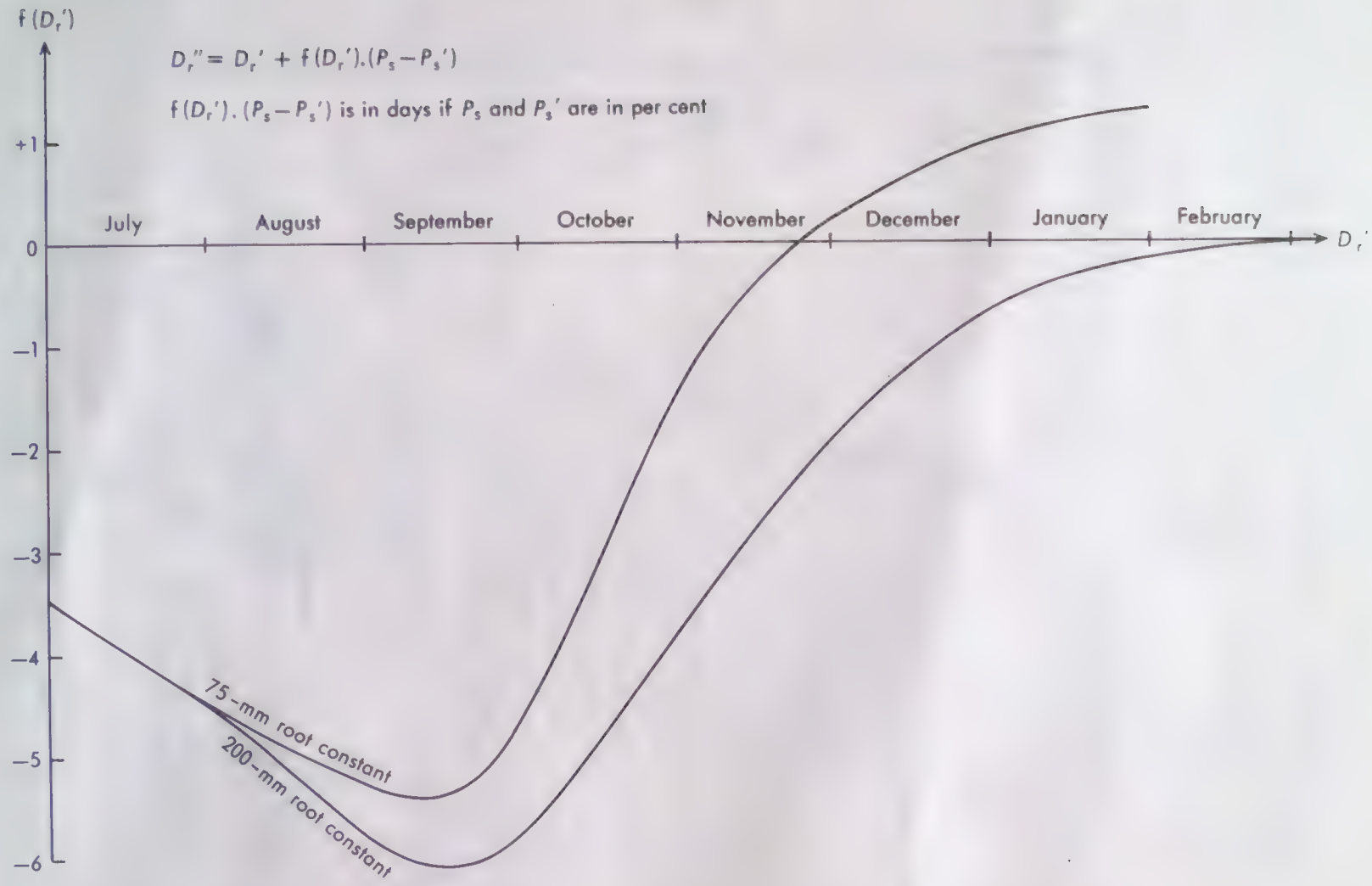

Figure 7. Adjustment of date of return to field capacity due to proportion of summer half-year rainfall

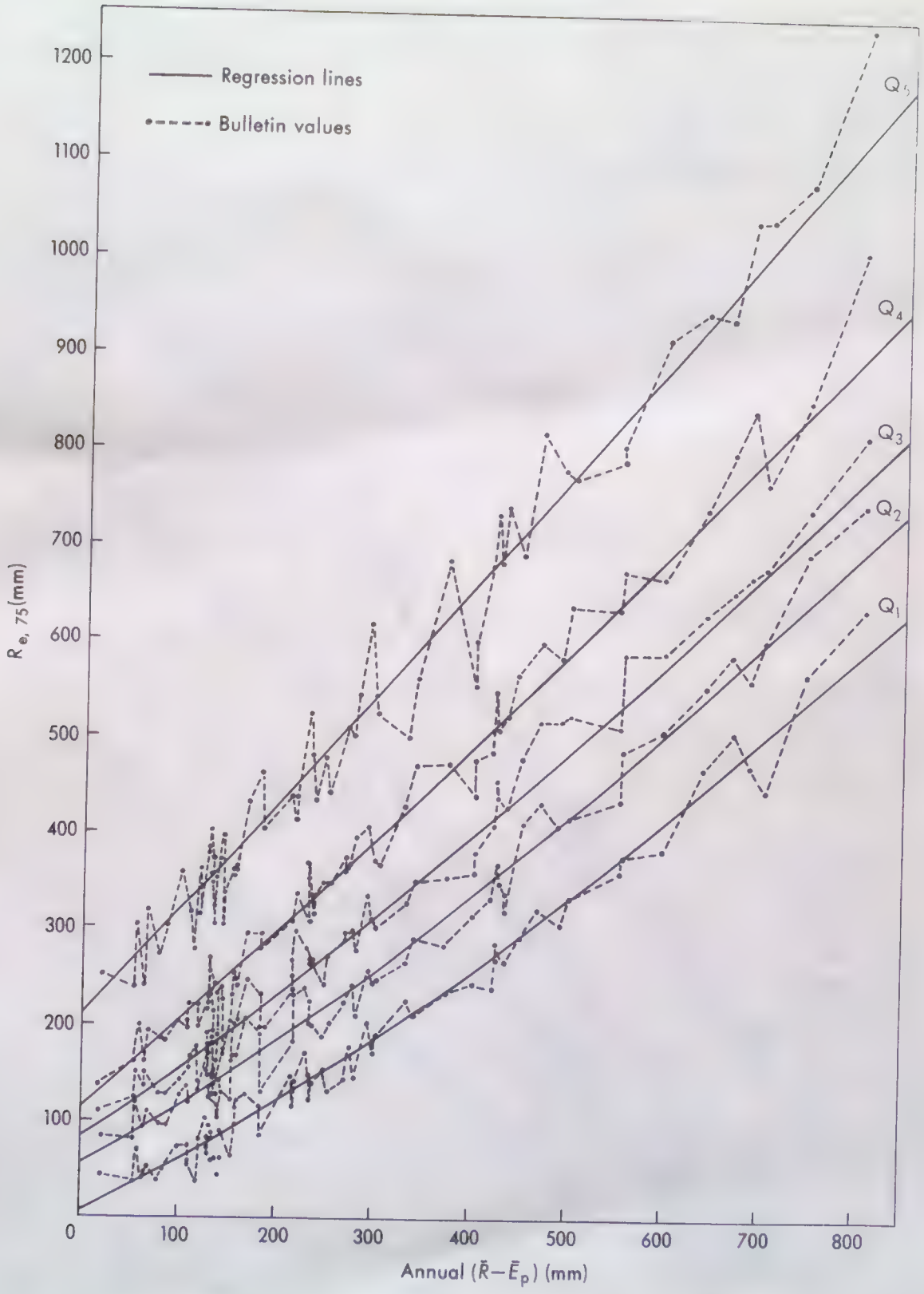

Figure 8. Annual hydrologically effective rainfall for a 75-mm root constant

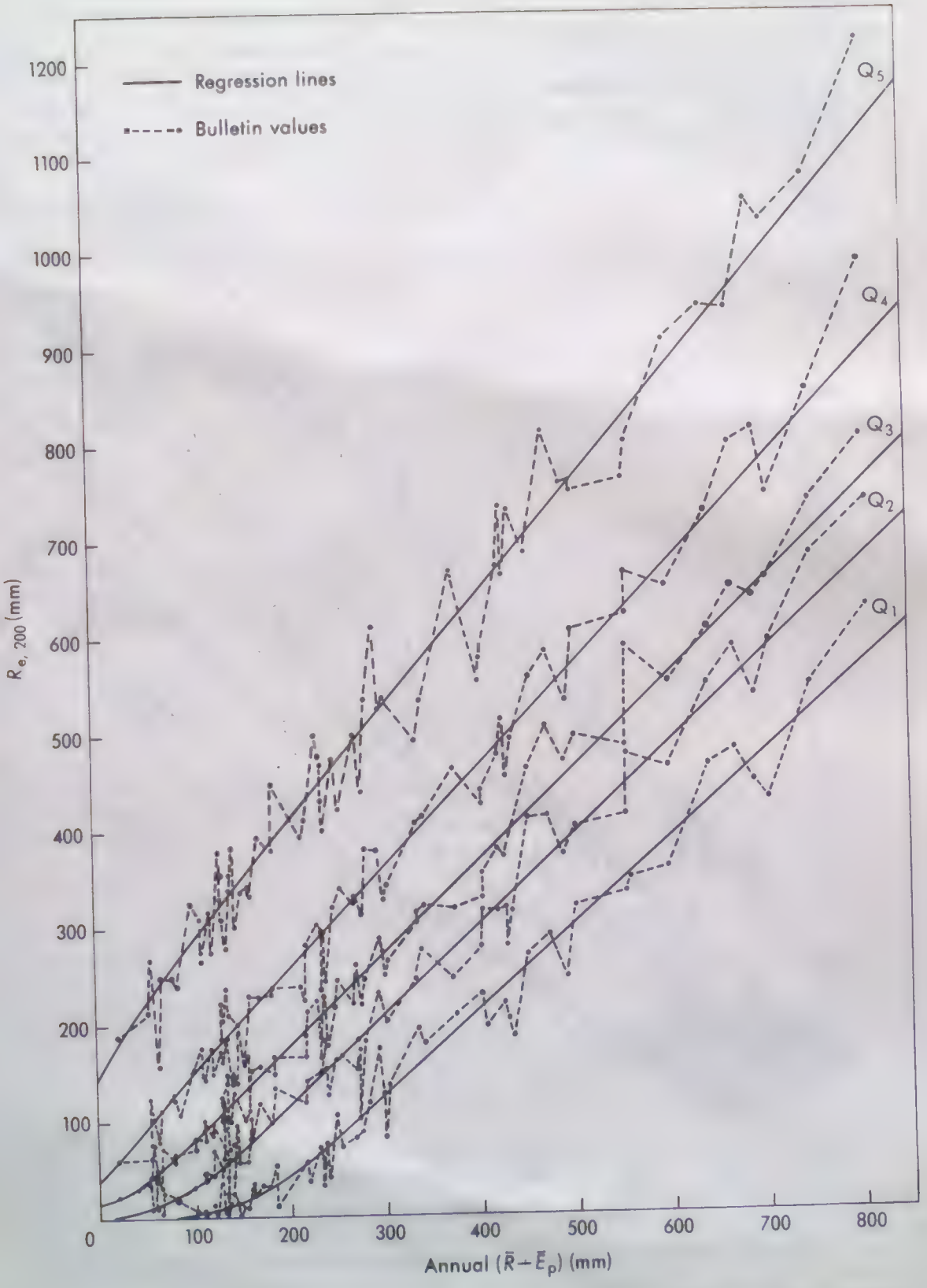

Figure 9. Annual hydrologically effective rainfall for a 200-mm root constant

TABLE 1 – EFFECT OF SEASONAL DISTRIBUTION OF RAINFALL ON DATE OF RETURN TO FIELD CAPACITY

Comparison between station A with rainfall uniformly distributed throughout the year, station B with a summer maximum of rain, and station C with a winter maximum of rain. One set of potential evaporation figures is used for all stations in both wet and dry years.

Month	Potential evapo-ration (mm)	Rainfall (mm)						End of month SMD (mm)					
		Dry year			Wet year			Dry year			Wet year		
		A	B	C	A	B	C	A	B	C	A	B	C
Apr.	50	40	40	40	70	70	70	10	10	10	0	0	0
May	75	40	47	33	70	83	57	45	38	52	5	0	23
June	90	40	47	33	70	83	57	95	81	109	25	7	56
July	90	40	47	33	70	83	57	145	124	166	45	14	89
Aug.	70	40	47	33	70	83	57	175	147	203	45	1	102
Sept.	40	40	40	40	70	70	70	175	147	203	15	0	72
Oct.	20	40	40	40	70	70	70	155	127	183	0	0	22
Nov.	5	40	33	47	70	57	83	120	99	141	0	0	0
Dec.	0	40	33	47	70	57	83	80	66	94	0	0	0
Jan.	5	40	33	47	70	57	83	45	38	52	0	0	0
Feb.	10	40	33	47	70	57	83	15	15	15	0	0	0
Mar.	25	40	40	40	70	70	70	0	0	0	0	0	0
Year	480	480	480	480	840	840	840						
Date of return to field capacity								31 Mar.	31 Mar.	31 Mar.	10 Oct.	1 Sept.	5 Nov.

TABLE 2 – ERRORS IN ESTIMATES OF LOSS OF, AND RETURN TO, FIELD CAPACITY

Root-mean-square number of days between bulletin (D_{rb}) and calculated dates (D_r' and D_r'').

	Loss of field capacity	Return to field capacity			
		75-mm root constant		200-mm root constant	
		based on annual $\bar{R} - \bar{E}_p$ ($D_{rb} - D_r'$)	taking into account P_s ($D_{rb} - D_r''$)	based on annual $\bar{R} - \bar{E}_p$ ($D_{rb} - D_r'$)	taking into account P_s ($D_{rb} - D_r''$)
Q_E	9	32	31	45	43
Q_{12}	7	15	11	16	12
Q_{23}	8	11	6	15	12
Q_{34}	6	9	7	16	14
Q_{45}	5	10	9	17	17
Q_L	9	17	17	19	19

TABLE 3 – PROBABILITY OF NON-RETURN TO FIELD CAPACITY FOR A 200-mm ROOT CONSTANT, RELATED TO ANNUAL ($\bar{R} - \bar{E}_p$)

Annual $\bar{R} - \bar{E}_p$ (mm)	0	40	20	120	160	200	240	280	320	360
Prob. of non-return (%)	45	31	22	14	9	6	3	1	0	0

TABLE 4 – RELATIONSHIP OF P_s, THE MEAN SUMMER HALF-YEAR RAINFALL/MEAN ANNUAL RAINFALL, TO ANNUAL $(\bar{R} - \bar{E}_p)$

Annual $\bar{R} - \bar{E}_p$ (mm)	0	100	200	300	400	500	600	700	800
P_s (%)	51.1	49.3	47.7	46.3	45.1	44.1	43.4	42.9	42.6

TABLE 5 – EXAMPLE OF DATE OF RETURN TO FIELD CAPACITY (75-mm ROOT CONSTANT)

Worthing, Sussex

Annual $(\bar{R} - \bar{E}_p) = 170$ mm; $P_s = 42.0\%$; $P_s' = 48.1\%$

	D_r' date	$f(D_r')$ days	$f(D_r').(P_s - P_s')$ days	D_r'' date	D_{rb} observed date
Q_E	11 Aug.	-4.8	+29	9 Sept.	16 Sept.
Q_{12}	13 Oct.	-3.3	+20	2 Nov.	25 Oct.
Q_{23}	7 Nov.	-1.0	+ 6	13 Nov.	6 Nov.
Q_{34}	22 Nov.	-0.1	+ 1	23 Nov.	11 Nov.
Q_{45}	18 Dec.	+0.8	- 5	13 Dec.	13 Dec.
Q_L	31 Jan.	+1.3	- 8	23 Jan.	6 Jan.

Rothamsted, Hertfordshire

Annual $(\bar{R} - \bar{E}_p) = 183$ mm; $P_s = 47.6\%$; $P_s' = 48.0\%$

	D_r' date	$f(D_r')$ days	$f(D_r').(P_s - P_s')$ days	D_r'' date	D_{rb} observed date
Q_E	8 Aug.	-4.8	+ 2	10 Aug.	3 Aug.
Q_{12}	11 Oct.	-3.7	+ 1	12 Oct.	5 Oct.
Q_{23}	4 Nov.	-1.3	+ 1	5 Nov.	28 Oct.
Q_{34}	21 Nov.	-0.2	0	21 Nov.	10 Nov.
Q_{45}	15 Dec.	+0.7	0	15 Dec.	3 Dec.
Q_L	28 Jan.	+1.3	- 1	27 Jan.	19 Jan.

TABLE 6 – QUINTILES OF $R_{e,75}$ AND $R_{e,200}$ FROM DATE OF LOSS OF FIELD CAPACITY UNTIL 1 JUNE (mm)

Values in parenthesis are values for the 200-mm root constant where these are different from those for the 75-mm root constant.

Annual $(\bar{R} - \bar{E}_p)$ (mm)	Quintiles					
	Q_1	Q_2	Q_3	Q_4	Q_5	Mean
0	0	0	0	1(0)	16(6)	3(1)
100	0	0	0	3(2)	23(21)	5
200	0	0	1	6	32	7
300	0	0	2	11	41	10
400	0	0	4	17	53	14
500	0	0	7	24	65	19
600	0	1	13	32	79	25
700	0	2	17	42	97	32
800	0	4	25	53	118	40

TABLE 7 – QUINTILES OF $R_{e,75}$ AND $R_{e,200}$ FROM 1 JUNE UNTIL DATE OF RETURN TO FIELD CAPACITY

Apr.–Sept. $(\bar{R} - \bar{E}_p)$ (mm)	Quintiles					
	Q_1	Q_2	Q_3	Q_4	Q_5	Mean
-180	0	0	0	0	1	0
-150	0	0	0	0	2	0
-120	0	0	0	0	5	1
-90	0	0	0	0	8	2
-60	0	0	0	0	12	2
-30	0	0	0	0	16	3
0	0	0	0	1	20	4
30	0	0	0	1	25	5
60	0	0	0	3	31	7
90	0	0	0	5	38	9
120	0	0	0	8	47	11

TABLE 8 – QUINTILES OF $R_{e,75}$ BETWEEN DATES OF RETURN TO, AND LOSS OF, FIELD CAPACITY

Annual $(\bar{R} - \bar{E}_p)$ (mm)	Quintiles					
	Q_1	Q_2	Q_3	Q_4	Q_5	Mean
0	6	54	79	109	204	91
100	57	112	148	197	311	165
200	114	176	222	286	416	243
300	176	245	298	375	523	323
400	242	320	378	466	630	407
500	314	399	461	559	738	494
600	389	480	547	652	845	583
700	466	563	631	747	950	671
800	544	644	716	839	1053	759

TABLE 9 – QUINTILES OF $R_{e,200}$ BETWEEN DATES OF RETURN TO, AND LOSS OF FIELD CAPACITY

Annual $(\bar{R} - \bar{E}_p)$ (mm)	Quintiles					
	Q_1	Q_2	Q_3	Q_4	Q_5	Mean
0	0	0	15	34	139	38
50	0	9	39	86	210	69
100	1	29	76	141	276	105
150	17	65	121	194	339	147
200	41	109	168	246	398	193
300	118	199	260	346	510	287
400	201	288	352	446	620	382
500	285	376	443	545	730	476
600	367	463	533	642	839	569
700	447	549	620	738	944	660
800	527	631	705	831	1048	749

TABLE 10 – RELATIONSHIP OF SUMMER HALF-YEAR $(\bar{R} - \bar{E}_p)$ TO ANNUAL $(\bar{R} - \bar{E}_p)$

Annual $(\bar{R} - \bar{E}_p)$ (mm)	0	100	200	300	400	500	600	700	800
Apr.–Sept. $(\bar{R} - \bar{E}_p)$ (mm)	-182	-143	-104	-65	-26	13	52	91	130

TABLE 11 – FOR A GIVEN ANNUAL $(\bar{R} - \bar{E}_p)$, ANNUAL $R_{e,75}$ INCREASES AS SUMMER $(\bar{R} - \bar{E}_p)$ DECREASES

G = Increase of annual $R_{e,75}$ (mm) corresponding to a 1-mm decrease of summer half-year $(\bar{R} - \bar{E}_p)$.

Annual $(\bar{R} - \bar{E}_p)$ (mm)	0	100	200	300	400	500	600	700	800
G (mm)	3.2	1.7	0.9	0.6	0.3	0.2	0.1	0.1	0.0

TABLE 12 – EXAMPLE OF CALCULATION OF QUINTILES OF HYDROLOGICALLY EFFECTIVE PRECIPITATION FOR THE WHOLE YEAR (75-mm ROOT CONSTANT)

	Worthing, Sussex						Rothamsted, Herts.					
Row		Q_1	Q_2	Q_3	Q_4	Q_5		Q_1	Q_2	Q_3	Q_4	Q_5
(a) Annual $(\bar{R} - \bar{E}_p)$	170						183					
(b) Quintiles from Fig. 8		102	164	207	270	397		110	173	217	282	414
(c) Summer ½-year $(\bar{R} - \bar{E}_p)$	-162						-116					
(d) Summer ½-year $(\bar{R} - \bar{E}_p)$ from Table 10	-116						-111					
(e) H = row (c) – row (d)	– 46						-5					
(f) G from Table 11	1.14						1.04					
(g) $G \times H$	-52						-5					
(h) Final quintiles = row (b) – row (g)		154	216	259	322	449		115	178	222	287	419
(i) Bulletin quintiles		129	209	248	296	346		117	197	236	281	403

TABLE 13 – ERRORS IN ESTIMATES OF HYDROLOGICALLY EFFECTIVE PRECIPITATION: ROOT-MEAN-SQUARE DIFFERENCES BETWEEN BULLETIN AND CALCULATED AMOUNTS (mm)

	Whole year		D_r to D_1		D_1 to 1 June	1 June to D_r
Quintiles	$R_{e,75}$	$R_{e,200}$	$R_{e,75}$	$R_{e,200}$	$R_{e,75}$ & $R_{e,200}$	$R_{e,75}$ & $R_{e,200}$
Q_1	20	28	20	28	0	0
Q_2	20	26	17	27	2	0
Q_3	20	24	22	24	5	0
Q_4	27	30	33	32	6	6
Q_5	40	43	35	48	11	10
Mean	19	22	20	23	3	2

TABLE 14 – COMPARISON OF PRESENT RESULTS WITH THOSE OF SMITH (1971)

Date of return to field capacity

Present work (75-mm root constant)			Smith (1971)	
Quintile boundary	Worthing	Rothamsted	Worthing & Rothamsted	Quintile boundary
Q_E	9 Sept.	10 Aug.	27 Aug.	Q_E
Q_{12}	2 Nov.	12 Oct.	12 Oct.	Q_{12}
Q_{23}	13 Nov.	5 Nov.	3 Nov.	Q_{23}
Q_{34}	23 Nov.	21 Nov.		
Q_{45}	13 Dec.	15 Dec.	30 Dec.	Q_{34}
Q_L	23 Jan.	27 Jan.	31 Jan.	Q_L

Hydrologically effective precipitation (mm)

Present work (75-mm root constant)			Smith (1971)	
Return to field capacity until loss of field capacity			Return to field capacity until 31 March	
Quintile	Worthing	Rothamsted	Worthing & Rothamsted	Quintile boundary
Q_1	149	109	50	Q_L
Q_2	209	168	150	Q_{12}
Q_3	252	213	220	Q_{23}
Q_4	311	275	290	Q_{34}
Q_5	436	403	410	Q_H